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MODELLING VOLATILITY OF PAKISTAN STOCK MARKET USING FAMILY OF GARCH MODELS

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ABSTRACT

The stock market across the globe is regarded as an essential component of modern economic systems. It is significant and acts as an indicator of a nation's economic health. A stock market is where the buying and selling of stocks takes place. Each market has an index: a group of stocks that are selected to represent the overall performance of that market. The economy is the sum of all the activities that go into making and spending money within a region or country. In the headlines, the economy is often measured and tracked through changes in Gross Domestic Product (GDP). Employment levels, the housing market, consumer confidence, and spending are other ways to measure the strength of the economy. We find the marginal distribution of all series by GARCH models with different error distributions. For the KSE 100 index, we select the ARMA (1,1) GARCH (1,1) with skewed student t distribution on the basis of the lowest AIC and BIC value. ARMA (1,1) model has the lowest AIC value and is therefore chosen for further analysis. We also checked from *auto.arima* function in R which also gives the same ARMA model. It also confirms that all coefficients of parameters give significant results. The parameter estimates of the volatility models are statistically significant. The sum of ARCH and GARCH coefficients are α_1 and β_1 are close to unity indicating that shocks to volatility have a persistent effect on the conditional variance, and also ensure stationarity. Further, the GARCH coefficient denoted by β_1 is high in the models which suggests that volatility is very sensitive to market shocks of KSE 100 index stock returns. The results of the ARCH-LM test are χ^2=138.49 ($p<0.01$) highly significant. ARMA (1,1) GARCH (1,1) with skewed student t distribution on the basis of lowest AIC and BIC value.

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INTRODUCTION

The most crucial pillar of every nation is its economy. Like all other sectors, a healthy economy is essential to reducing the need for a developed, powerful, independent, and secure nation. Whether it is defense, health, education, food, housing, or social security, all of society's needs are reliant on the nation's economic resources. All of them rely on abundant economic resources. Any nation's economic standing has an impact on its sovereignty. The battles of today are commercial conflicts, conducted in trade markets rather than on open fields. Despite its pivotal role, the Pakistan Stock Exchange faces challenges, including market volatility, regulatory complexities, and external economic pressures. However, these challenges present opportunities for continued growth and improvement (Audi et al., 2022). There are several macroeconomic factors which affect the stock market performance. In this study, we explore the dependence structure of Pakistan's stock market with crude oil prices and exchange rates. Pakistan is a developing nation that imports oil and is dealing with many economic crises as a result of growing inflation and unpredictability in the world economy. One of the primary causes of the market's occasionally odd behavior is political instability. Pakistan's currency rates vary more quickly than those of other industrialized nations due to its status as a developing nation (Siddiqui et al., 2023).

This study focuses on the Pakistan stock market. We selected the main Stock market index which is KSE 100 index. We have studied stock market return, by taking weekly closing prices followed by the descriptive statistics and then converting it into returns, checking their stationarity, identifying of mean equation by ARMA specification, and identifying of marginal model by using GARCH models, as well as finding the most suitable bivariate copula GARCH model.

Pakistan Stock Exchange

The stock market is a vital area of the country's economy. Pakistan Stock Exchange was established on September 18, 1947. Then it was formally named as "Karachi Stock Exchange" on March 10, 1949. To meet the need at the provincial level Lahore Stock Exchange was established in October 1970. Then for the investors from northern areas Islamabad Stock Exchange was established in October 1989. All three stock exchanges had separate indices and management interfaces. On January 11, 2016, by the Act 2012 (corporatization, mutualization, and integrating) government decided to integrate its operations under the new name Pakistan Stock Exchange. The stock market is one of the main indicators of any economy. It reflects the volume of trade, preferences, and approach of the people of the country (Zaffar and Hussain, 2022). A base of 1000 points was initially used for the KSE 100 index. The first ten years of its existence were not very noteworthy because the graph fluctuated between 900 and 2000 points. It reached 2700 points by December 2002, after which it progressed slowly and upwards gradually. The KSE 100 index had a significant increase of 47.2% during the 2004–05 fiscal year (from 5279 points to 7770 points), and the market capitalization showed a growth of 55.8%, which helped the Karachi stock market rise to the fifth rank in the global market. The improvement led to the international magazines "Business Week" and "USA Today" praising the Karachi stock market as the world's finest performing market. In March 2006, it reached its highest point total of 11485. But in this case, during the period of steady advancement, the KSE 100 index saw notable declines at specific moments. For example, the May 2006 stock market meltdown caused the index to lose 1,500 points. Following this decline, the KSE 100 index managed to regain its previous position, and on April 20, 2008, it hit its highest peak in history, which was 15,000 points. It barely remained at this level for two months until the Karachi stock market saw a very serious fall in August 2008, during which the KSE100 index dropped ten thousand points. During the fiscal year 2009–2010, it made up for the lost ground, and on November 7, 2012, it once more reached a record-breaking high of 16218 points in 2011–12. This kind of development led to it being named Asia's "Best Emerging Market." The KSE 100 index reached an amazing record level of 23,097 points on June 15, 2013. After the merger of stock markets in 2016, the stock market faced ups and downs due to economic events, geopolitical developments, and global market trends that may have influenced the performance of the Karachi Stock Exchange during the last years (Sohail and Javid, 2014).

The stock market is the ideal platform both for investors and entrepreneurs to trade their stocks. It works as a production house for generating equity, and capital, and creating new ventures, and layouts. It provides an opportunity for a common man to participate and get ownership in a multi-billion Worth Company. It's also working as a secondary market for trading commercial papers and government bonds (Pandya et al., 2024).

The second-biggest nation in South Asia is Pakistan. There are more than 220 million people living there. The main objective of PSX is to provide a safe, reliable, and efficient marketplace for investors. Where they can safely buy or sell shares of listed companies and securities.

Significance of the Study

For researchers, the relationship between crude oil prices, exchange rates, and stock exchanges is crucial since it is commonly acknowledged to lead to changes in the expansion of the stock market. Monetary managers consider the underlying relationship between crude oil prices, currency rates, and stock exchange when formulating contributing policies. It is intended that this work will inspire Pakistani researchers and academics the courage to use improved methods in their respective domains.

Necula (2010) focused on the index returns from four markets to analyze the dependency structure using the Copula-GARCH model. The data used in the study consists of daily returns from stock indexes in Eastern European emerging markets (Czech Republic and Hungary) and developed financial markets (Germany and the USA). Dajcman (2013) examined the dependence structure between the returns of Croatian and five European stock markets by using the copula GARCH technique. The study suggests that the relationship between Croatian and major European stock markets is dynamic and can be adequately characterized by dynamic normal or symmetrized Joe-Clayton copula GARCH models. This study concludes that the lower tail dependence and upper tail dependence are the same in pattern. Zhu et al. (2014) explored the asymmetric dependence between crude oil prices and Asia Pacific stock returns. They used the constant and time-varying copulas to evaluate the tail correlations. They divided the data into two periods, before and after the financial crisis. They found that the time-varying copula is a good fit on the basis of the lowest AIC value. Hamma et al. (2018) examined the dependency between oil prices and the stock markets of Tunisia and Egypt. ARMA-GARCH is used to find the marginal densities of series. Extreme value copula is used for joint distribution to investigate the tail's behaviors and changes over time. This study revealed that there exists significant and symmetric tail dependence.

Karakas (2016) used the copula GARCH method to examine the dependence structure between crude oil and the stock markets of Turkey. This study proposed that copulas are best in describing complex multivariate dependence structures that are non-linear and tail-dependent. They combined the ARMA, ARCH, GARCH, EGARCH, and GJRGARCH models to find the marginal distribution of the return series. Six copula families are used to find the tail dependence. Kanwal and Khan (2021) studied the dependence structure between the EUA (European Union Allowance) and four energy commodities (crude oil, coal, natural gas, and ethanol). The marginal densities are obtained by the ARMA-GARCH student-t model. Then different bivariate copulas are used to capture tail dependence between carbon market and energy commodities. This study found that EUA has more dependence on oil and coal. Jafry et al. (2022) explored the dependence structure between the conventional and Islamic stock indices of Malaysia using Copula GARCH models. This study used daily data covering a period from May 2007 to September 2018. The optimal marginal models involve AR (1) for conditional mean estimation. Additionally, GARCH (1,1) and EGARCH (1,1) models effectively capture the conditional variance aspect. Among the considered models, AR (1)-EGARCH (1,1) with t distributions stands out as the preferred choice for the pair, based on the lowest information criterion value.

Kimani et al. (2023) investigated the dependence structure among four cryptocurrencies using the copula GARCH model. Prices of four cryptocurrencies (Bitcoin, Binance, Lite coin, and Dogecoin) were analyzed. The findings demonstrated that basic GARCH (1,1) under the very adaptable ARMA-GARCH model was suitable for determining the actual patterns of index returns. The most effective method for determining dependency between the cryptocurrencies is Standard GARCH (1,1). When the copula standard GARCH (1,1) model was fitted to the currencies, it was found that, out of the chosen currencies, the pair Bitcoin and Litecoin had the largest tail dependency. Cryptocurrency market, which suggests that fluctuations in Litecoin's price would impact fluctuations in Bitcoin's price and vice versa. Dogecoin has the best optimization among the cryptocurrencies, according to optimization results. The study's findings show that, despite the strong link between Litecoin, Bitcoin, and Binance, investing in Dogecoin considerably lowers risk.

METHODOLOGY

We use GARCH modeling to model the marginal distribution of each time series. We use copula GARCH models to find the dependence structure among the Pakistan stock exchange.

Description of data

Pakistan's economy is significantly impacted by the Karachi Stock Market. In this sense, investigating stock exchange performances at the 100 index might provide valuable insights into the nation's economic situation. Returns on stock exchanges have a variety of macroeconomic aspects; so, one must research these elements in order to understand the stock returns. We have chosen the crude oil prices (OP) and exchange rate (EX) for this investigation. This study uses weekly data on these factors, spanning the timeframe from January 2000 to December 2022. The websites investing.com and yahoo finance.com provide the weekly data. The available data of crude oil prices are in USD per barrel, to bring the consistency in crude oil prices we have to convert the prices in PKR. As we have KSE 100 index and exchange rate are also in PKR.

Calculation of Returns

Raw price series are typically transformed into return series since working directly with the price series is not preferred for several statistical reasons. Aloui and Aissa (2016) suggest calculating the returns of each time series by using the following formula

$$r_t = \ln (p_t / p_{t-1}) \quad (1)$$

Where r_t is the return at time t, ln is the natural logarithm, p_t is the closing price on a given day and p_{t-1} is the closing price on the previous day.

Descriptive Statistics

To check the distributional properties of the return series during the study period, some descriptive statistics are described in detail. The statistics include mean return, standard deviation, skewness, kurtosis, and the Jarque Bera test for normality.

Stationarity Test

The stationarity of the series is crucial when utilizing the time series model; otherwise, it might cause a lot of issues and have unfavorable effects. We employed the augmented Dickey-Fuller test (ADF) for the unit root and the Phillips-perron (PP) test for the unit root as statistical techniques to determine whether or not our data is stationary. The most often used statistical test for determining whether a series is stationary is the Dickey and Fuller (1979) enhanced Dickey-Fuller test.

Model for Marginal Distribution

To find the bivariate copula GARCH models, the first step is to find the univariate marginal distribution for each return series. The copula GARCH model allows us to describe the potential skewness and leptokurtosis of each return series. We model each return series by the ARMA-GARCH model and transform the residuals into the uniform distribution of each selected model. After that, we use bivariate Archimedean and elliptical copula families to capture the dynamic dependence structure.

Autoregressive Moving Average Model

Wold (1938) integrated the AR(p) and MA(q) models, presenting a generalized version that combines autoregressive (AR) and moving average (MA) components. He demonstrated that ARMA (p, q) processes provide a versatile framework for modeling all stationary time series, where the selection of suitable orders, "p" for the number of AR terms, and "q" for the number of MA terms, allows for effective representation. The ARMA model can be expressed as:

$$r_t = \theta_0 + \sum_{i=1}^{p} \alpha_i r_{t-i} + \sum_{j=1}^{q} \beta_j \epsilon_{t-j} + \epsilon_t \quad (2)$$

Here, θ_0 represents the constant term, α_i signifies the parameter associated with the autoregressive component of order p, βj denotes the parameter linked to the moving average component of order q, and ϵt stands for the error term at time t. When estimating the ARMA parameters, certain properties are attributed to the error term, such as ensuring that the mean of ϵt is zero and the variance is σ^2. While the first assumption of a zero mean is readily satisfied in financial time series data, the assumption of constant variance is often not met, leading to the challenge of heteroscedasticity. This issue can be addressed by employing a family of ARCH models.

Box-Jenkins Approach for ARMA Model Specification: While the ARMA model was already established, Box-Jenkins (1976) pioneered the systematic estimation of this model, involving a three-step process:

1. Identification
2. Estimation
3. Diagnostic checking

Model Selection Criteria: Examining the autocorrelation and partial autocorrelation plots provides a preliminary indication for selecting the number of parameters. Refining the choice for the most accurate number of parameters involves selecting values that minimize information criteria i.e. AIC.

Akaike's Information Criteria: Akaike (1974) proposed the method used for the comparison of different models and chose the one with the lowest value of AIC.

$$\text{AIC} = 2\text{K} - 2\ln(\text{likelihood}) \quad (3)$$

Where k = no of parameters to be estimated in the model and the second one is a log of the likelihood function of the model.

Residual Diagnostics

Following the execution of the selected ARMA model, the subsequent step involves analyzing the residuals. Plotting the residuals helps identify outliers, detect any discernible patterns, and, crucially, ascertain the presence of ARCH effects. While graphical methods provide an initial insight, they are informal and lack authenticity. To rigorously test the presence of ARCH effects in the return series residuals, the ARCH-LM test is employed.

ARCH (q) Model

The ARCH model is the most often used non-linear model in finance to represent volatility. The primary source of inspiration for the Autoregressive Conditional Heteroscedasticity Model (ARCH model), initially introduced by Engle (1982), is the non-constant volatility observed in financial time series. According to Engle's model, the variance of the residuals at time t is reliant on the squared error terms from earlier periods.

In general ARCH (q) model will be given by:

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \quad (4)$$

Estimated coefficients that are α_o and α_i must be non-negative, otherwise, it would be meaningless.

GARCH (p, q) Model

The ARCH model has inherent limitations, notably in determining the appropriate number of lags for the squared residuals to capture the dynamics of conditional variance. Selecting a higher ARCH order becomes necessary. To address these challenges, Bollerslev (1986) introduced an extension of the ARCH model known as the GARCH model. In this model, Bollerslev proposed that the variance of residuals at time "t" is dependent on lagged square error terms and lagged conditional variances.

In general, the GARCH (p, q) model is given as:

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (5)$$

Where α_0, α_i, and β_j are non-negative, with $\alpha_i + \beta_j < 1$.

A simpler form of GARCH (p, q) model is GARCH (1,1) model which is given as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (6)$$

Here, α_0 denotes the long-term average value, α_1 provides an estimation of volatility information from the previous period, and β_1 measures the persistence effect. Typically, a GARCH (1,1) model suffices to capture the renowned volatility clustering phenomenon.

RESULTS AND DISCUSSIONS

The data used in this study is the weekly closing prices of the Pakistan stock market. We select the major baseline index of the market which is the KSE 100 Index. We take the weekly closing prices of exchange rates and crude oil. We convert the crude oil prices into rupees per barrel to make the unit scale consistency between all three series. We analyze these series one by one. Firstly, we model the marginal distribution of the KSE 100 index by ARMA-GARCH model with six error distributions then we select the best marginal distribution. Then we find the marginal distribution of crude oil and exchange rate in the same way. After that, we use the standardized residuals of selected marginal distributions to model the different copula families. Then we transformed these standardized residuals into uniform distribution by integral probability transformation. We use a bivariate copula GARCH model for all three pairs.

Marginal Distributions of KSE

We are using weekly closing prices of the KSE 100 index, for different statistical reasons it is not preferable to directly work with the prices, so mostly raw price series data is firstly converted into return series. So, we have to convert our data into returns by using logarithm transformation.

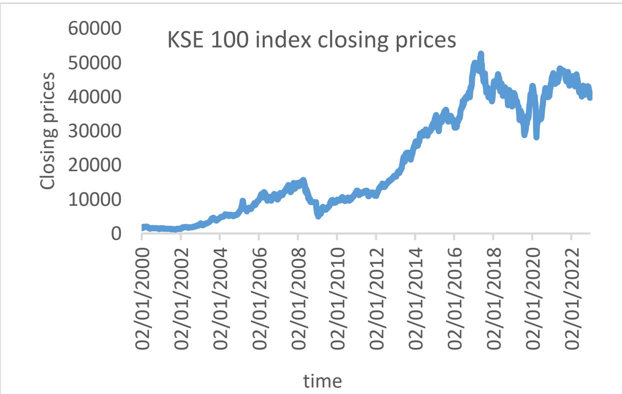

Figure 1. KSE 100 index closing prices.

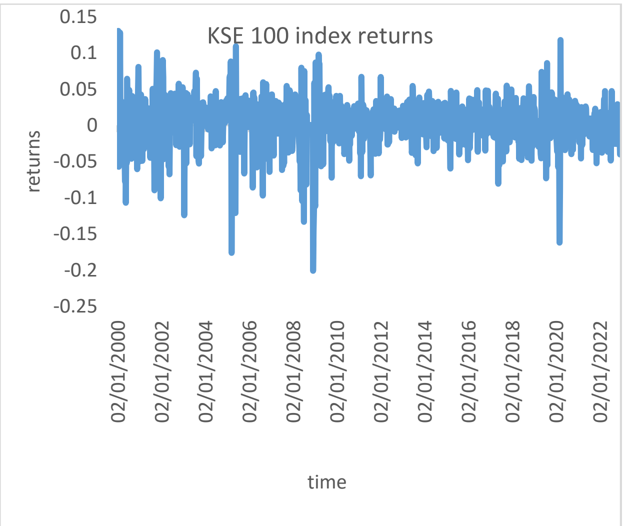

Figure 2.KSE 100 index log returns.

Table 1. Descriptive statistics of KSE 100 index returns.

N	Minimum	Maximum	Mean	St. Deviation	Skewnes	Kurtosis	JB p-value
1200	-0.2009	0.13034	0.0028	0.0312	-0.8737	4.98937	0.000

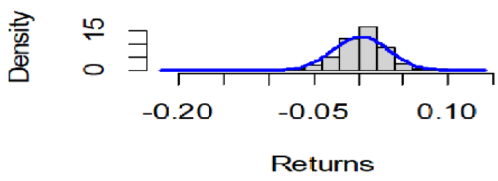

Figure 3. Histogram of KSE 100 Index Stock Returns.

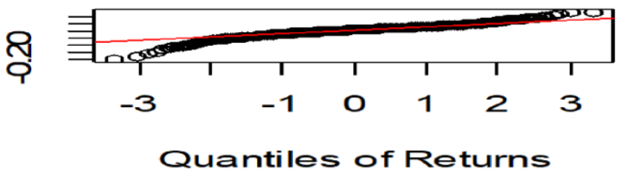

Figure 4. Q-Q plot of KSE 100 Index Stock Returns.

Descriptive statistics of KSE 100 index stock returns are presented in Table 1. The positive mean indicates that return increases over time in Figure 3. The stock returns are negatively skewed implying that the tail on the left side of the distribution is thicker and does not follow normal distribution. The kurtosis value is higher than the standard value 3, indicating that the stock return is attributed to a fat tail characteristic. It indicates occasional extreme positive and negative returns. Furthermore, the p-value from the JB test indicates that the distribution of returns is not normal.

In Figure 4 normal Q-Q plot and histogram of returns also show that the stock returns do not follow the normal distribution and confirm the stylized fact of volatility which leads us to use the GARCH model for the marginal distribution.

Testing for stationarity

To check whether the daily stock returns of the KSE 100 index are stationary or not, in Figure 1. We use use graphical presentation which shows that it is not stationary and again in Figure 2 we display the first difference form. Now we follow statistical procedures that are the ADF test for unit root and the PP test for unit root. From Figure 5 and Figure 6, it can be seen that our data represents stationarity but for further clarification, we use the following unit root tests. The results of the ADF test and PP test have been given below.

Table 2. Unit Root test for KSE 100 Index Stock Returns.

Unit Root Test	t-statistic	p-value
ADF TEST	-10.907	0.01
PP TEST	- 1099.1	0.01

The results of the ADF test and PP test presented in Table 2, show that the return series of the KSE 100 index is stationary and has no need for any transformation, so we further proceed with our analysis.

Mean Equation Selection for KSE 100 Index Stock Returns

ARMA model is used for conditional mean equation, mean equation eliminates the problem of autocorrelation which is presented in the volatility model. Different techniques have been proposed and investigated in the literature, yet numerous practitioners generally follow Box-Jenkins's approach. By observing the plot of autocorrelation and partial autocorrelation rough idea of p and q can be obtained. ACF plot is used to determine the order of q and PACF plot is used to determine the order of p. The more accurate value of the parameters p and q was picked up by choosing the lowest values of AIC among several experiments.

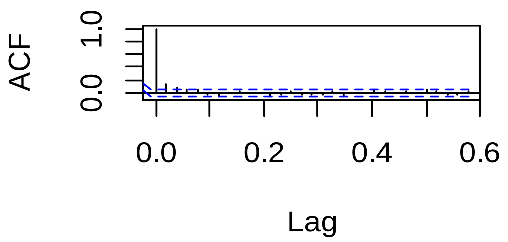

Figure 5. ACF plot of KSE 100 Index Stock Returns.

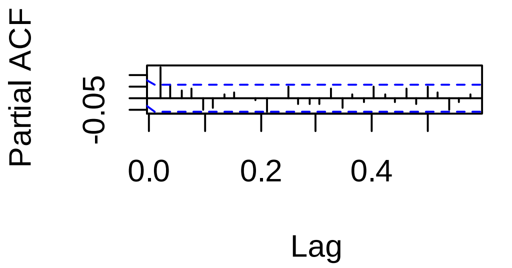

Figure 6. PACF plot of KSE 100 Index Stock Returns.

Table 3. Evaluation of different ARMA models.

Model	AIC
(1,1)	-4932.24
(0,1)	-4925.41
(1,0)	-4928.02
(1,2)	-4930.32
(2,1)	-4930.31
(2,2)	-4929.41
(2,0)	-4930.70
(0,2)	-4928.71

Table 3 shows that ARMA (1,1) model has the lowest AIC value and is therefore chosen for further analysis and also confirmed by Figure 5 and Figure 6. We also checked from auto.arima function in R which also gives the same ARMA model. It also confirms from Table 4 that all coefficients of parameters give significant results. Hence ARMA (1,1) model is adequate and used for further analysis.

Table 4. Estimation of parameters of ARMA (1,1) model.

Estimation	Estimated values	Std. Error	p-value
AR1	0.5756	0.1211	0.00000
MA1	-0.4479	0.1315	0.00068
Intercept	0.0028	0.0012	0.01

Residual Diagnostics

After running the desired ARMA model the next step is to run residuals. Plot the residuals to see whether our data have any outlier, any predictable pattern, the absence of serial correlation among residuals, and most importantly the presence of the ARCH effect. For the best-fitted model, the residuals should not be serially correlated, to check it out we have fitted the Ljung-box test. ARCH-LM test is used for testing the presence of the ARCH effect.

Ljung-Box Serial Correlation Test

To determine the absence of serial correlation in the residuals of the return series Ljung-Box test is used, and the result confirms the absence of serial correlation.

Table 5. Ljung-Box serial correlation test.

Q Statistic	Df	P-value
8.7921	10	0.5519

The null hypothesis of the Ljung-Box test is that there is no autocorrelation in the residuals at different lags. In Table 5. the p-value of 0.5519 is greater than a significance level (0.05) so we do not reject the null hypothesis and conclude that the residuals may exhibit white noise characteristics.

Lagrange Multiplier Test to Detect the ARCH Effect

To get the presence of conditional heteroscedasticity in the residuals of the return series ARCH-LM test is used. The null hypothesis stated that there is no conditional heteroskedasticity present in the time series data.

Table 6. ARCH-LM test to detect ARCH effect.

Chi-squared	df	P-value
138.49	12	0.0000

The results of ARCH-LM test presented in Table 6 conclude that the test statistic is highly significant confirming the presence of ARCH effect in the residuals of the time series model, now the next step is to run a family of GARCH models.

Results of GARCH Models for KSE 100 Index

We use R software (library rugarch) to find out the best-fitted model of GARCH models with different error distributions are normal distribution, skewed normal distribution, t-distribution, skewed-t distribution, generalized error distribution, and skewed generalized error distribution. Table 7 and Table 8 define the results of all six models and diagnostics respectively. As we have calculated the parameter estimates of conditional mean and conditional variance equations, it can be seen that all ARMA terms are significant

Table 7. Results of GARCH (1,1) UNDER different error distributions.

ARMA (1,1)-GARCH (1,1)	Norm	Snorm	std	sstd	ged	Sged
Mu	0.002939	0.002302	0.004717	0.003005	0.004743	0.002740
	(0.000604)	(0.005508)	(0.00000)	(0.001057)	(0.00004)	(0.003062)
AR1	0.310303	0.265709	0.518161	0.475585	0.521072	0.446351
	(0.009331)	(0.222052)	(0.000029)	(0.000790)	(0.00000)	(0.367892)
MA1	-0.87902	-0.168265	-0.371429	-0.338938	-0.383246	-0.310883
	(0.317678)	(0.452881)	(0.005505)	(0.025261)	(0.00000)	(0.537312)
Ω	0.00002	0.000037	0.000119	0.000121	0.000130	0.000135
	(0.002721)	(0.020844)	(0.000170)	(0.000104)	(0.000198)	(0.000007)
α_1	0.094854	0.113507	0.226775	0.236550	0.200662	0.223850
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00005)	(0.00000)
β_1	0.884946	0.846496	0.654296	0.639842	0.675949	0.631943
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Skew		0.775340		0.822188		0.792245
		(0.00000)		(0.00000)		(0.00000)
Shape			5.172346	5.702195	1.220324	1.272599
			(0.00000)	(0.00000)	(0.00000)	(0.00000)

Table 8. Results of GARCH (1,1) Diagnostic tests for KSE 100 Index Stock Returns.

GARCH (1,1)	Norm	Snorm	std	sstd	ged	sged
AIC	-4.2857	-4.3269	-4.3945	-4.4124	-4.3743	-4.4025
BIC	-4.2603	-4.2972	-4.3648	-4.3785	-4.3446	-4.3686
ARCH-LM	0.9163	0.9799	0.9947	0.9940	0.9959	0.9949
Logliklihood	2577.438	2603.119	2643.673	2655.437	2631.553	2649.505

We have calculated parameter estimates of conditional mean and conditional variance equations, it can be seen that all ARMA terms are significant under t-distribution, skewed t-distribution, and generalized error distribution which means the mean equation is appropriately identified. The parameter estimates of the volatility models are statistically significant. The sum of ARCH and GARCH coefficients are α_1 and β_1 are close to unity indicating that shocks to volatility have a persistent effect on the conditional variance, and also ensure stationarity. Further, the GARCH coefficient denoted by β_1 is high in the models which suggests that volatility is very sensitive to market shocks of KSE 100 index stock returns. Results of diagnostic tests for KSE 100 index stock returns disclose that ARCH effect which was present earlier has been removed after fitting GARCH models under different error distributions as the probability of ARCH-LM test is all greater than 0.05, which implies that volatility models are well specified. Furthermore, both model selection criteria tests such as AIC and BIC have been obtained for all the candidate models and the highest log likelihood value for all models is also calculated. According to these information criteria, the best fitted model is ARMA (1,1)-GARCH (1,1).

Table 8 defines the results of fitting GARCH (1,1) model with different error distributions of Exchange rate returns. We have calculated parameter estimates of conditional mean and conditional variance equations, it can be seen that the model with Generalized error distribution suggests that the mean, ARCH, and GARCH coefficients are significant, indicating the presence of certain dynamics in the volatility process. The parameter estimates of the volatility models are statistically significant. The sum of alpha and beta close to one indicates that both recent shocks (captured by alpha) and past volatility (captured by beta) contribute significantly to the current volatility. The volatility process is persistent, and shocks have a lasting impact on future volatility.

Results of diagnostic tests for exchange rate returns disclose that ARCH effect which was present earlier has been removed after fitting GARCH models under different error distributions as the probability of ARCH-LM test is all greater than 0.05, which implies that volatility models are well specified. Furthermore, both model selection criteria tests such as AIC and BIC have been obtained for all the candidate models and the highest log likelihood value for all models is also calculated. According to these information criteria, the best-fitted model is ARMA (3,2)-GARCH (1,1) with generalized error distribution.

CONCLUSIONS AND RECOMMENDATIONS

The weekly data of the stock price KSE-100 index, the price of WTI crude oil, and the US dollar per rupee exchange rate were obtained for a twenty-two-year period from January 01, 2000, to December 31, 2022. The data set for each variable consisted of a total of 1200 observations. The results based on these time series were analyzed by using R Studio and Excel software. Firstly, the descriptive analysis of each return series was performed. After checking the stationarity, the mean equation is modeled by the ARMA process. The general guess about the orders of p and q are determined by the ACF and PACF plots. All candidate ARMA models are evaluated and select the model with minimum AIC value. The residual diagnostic tests, Ljung box test, and Lagrange multiplier test are performed. Ljung box test found that there is no autocorrelation and concluded that the residuals may exhibit white noise characteristics. Due to the presence of the ARCH effect, each return series modeled by the GARCH (1,1) model with six types of error distributions such as normal distribution, skewed normal distribution, Student-t distribution, skewed student-t distribution, generalized error distribution, and skewed generalized error distribution. The adoption of ARMA-GARCH is motivated by the stylized effects of our data including the volatility clustering and serial dependence. In the process of modeling the dynamic dependence structure, the first step is to find the appropriate marginal distribution for each return series. For the KSE 100 index, we select the ARMA (1,1) GARCH (1,1) with skewed student t distribution on the basis of the lowest AIC and BIC value.

In this study, we use weekly prices as sampled data, while in future research data samples can be taken on a daily and monthly basis. Further research may also be conducted by using other copula families like the vine copula, C-vine copula, and D-vine copula. We use standard GARCH to model marginal distribution but for future studies, EGARCH, TGARCH and APRACH, etc. can be used. Future studies may also be carried out for other lag orders that can be taken for e.g., GARCH (1,2), GARCH (2,1), etc.

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